A Level Maths – Preparation for sixth form

A Level Maths is a subject that is challenging and rewarding. It will take hard work and perseverance, yet the journey of understanding and career options it opens are worth the commitment.

You must complete every section of the *compulsory* preparation work needed to support your transition into Maths at A level, each section contains different elements of a GCSE course that are essential to access the A Level course. Each section has examples, video links, practice questions, and answers. There are extension questions in each section to really push your understanding.

Make sure you complete the summer assignment as this will be collected in during the first week of the course, there will also be a transition assessment around three weeks into the course based on the summer assignment to show your understanding of the content. This allows us to support you in being successful in the course.

It is also highly recommended that you consider some of the *suggested* preparation work. These resources will give you the opportunity to look at Maths in a wider context and give you a sense of the joy that can come from the subject. These resources are listed towards the end of the document.

		Description	Completed
Compulsory preparation work	Section 1 Manipulating algebra	Algebra is the cornerstone of A Level Mathematics, to find success you need to make sure you can manipulate it easily.	
	Section 2 Factorising and completing the square	Factorising can vary in difficulty and there are numerous methods you can use. Make sure you are fluent at factorising algebra of all the various forms in this section. Completing the square is more of a side note in GCSE courses, you will experience it's full power at A Level.	
	Section 3 Solving and Sketching Quadratics	During your first year of A Level Maths many questions end up with solving a quadratic, make sure you are all over this topic. Sketching quadratic helps to give you a visual understanding of the algebra, this will be very important during the applied side of the A Level Maths course.	
	Section 4 Simultaneous equations	Simultaneous equations are a GCSE topic that features throughout the A Level Maths course. They will get more challenging so make sure you are on top of these.	
	Section 5 Inequalities and rearranging equations	Inequalities will be revisited early in the A Level Maths course and will introduce some new notation to go with them, ensure you can solve all the ones in this section. Rearranging equations is an essential skill you will use in most lessons.	
	Section 6 Linear Graphs	Graphs allow us to understand algebra visually, during the course you will see aspects of mathematics applied in contexts. Understanding linear graphs will be essential to allow you to understand these topics.	
	Summer assignment	Section A – This is a must for all students to complete, focusing on GCSE skills essential for A Level Maths Section B – This is more challenging and focusses on the GCSE skills required for Further Maths. If you are studying A Level Maths and want a challenge, go for it.	
Suggested preparation	Documentaries, videos, and talks	There is so much out there, here are a few links to a few favourites. (See the contents on the next page to find these resources)	
	Podcasts	Take a walk and listen to some of these gems.	
	Books	There are many great books on Maths, here are a few to we have picked.	
	Films	Get the popcorn ready!	

If you want any advice or encounter difficulties email Mr Newton knewton@ecgbert.sheffield.sch.uk

Contents

Section 1 – Manipulating Algebra

Expanding brackets and simplifying expressions

Key points and examples [Video link]

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

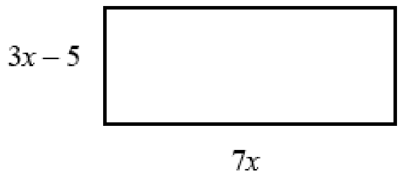
6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **b** $\left(x+\frac{1}{x}\right)^2$

Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

Surds and rationalising the denominator

Key points and examples   Video link

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\dfrac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\dfrac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$ $= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \frac{1 \times \sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9} = 3$
$= \frac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$ (We multiply by this 'conjugate' as it removes the surds using the difference of two squares)
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1
$= 3\sqrt{5}-6$	

Practice – Without using a calculator

Hint

One of the two numbers you choose at the start must be a square number.

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

Watch out!

Check you have chosen the highest square number at the

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

c $4\sqrt{3}$ **d** $5\sqrt{7}$

e $10\sqrt{3}$ **f** $2\sqrt{7}$

g $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$

c $3\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

c $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

e $\sqrt{2}$ **f** $\sqrt{5}$

g $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

Key points and examples

[Video link]

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$ $= \dfrac{1}{16}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{\left(2x^2\right)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

Answers

1	**a**	1	**b**	1	**c**	1	**d**	1
2	**a**	7	**b**	4	**c**	5	**d**	2
3	**a**	125	**b**	32	**c**	343	**d**	8
4	**a**	$\frac{1}{25}$	**b**	$\frac{1}{64}$	**c**	$\frac{1}{32}$	**d**	$\frac{1}{36}$

5	**a**	$\frac{3x^3}{2}$	**b**	$5x^2$
	c	$3x$	**d**	$\frac{y}{2x^2}$
	e	$y^{\frac{1}{2}}$	**f**	c^{-3}
	g	$2x^6$	**h**	x

6	**a**	$\frac{1}{2}$	**b**	$\frac{1}{9}$	**c**	$\frac{8}{3}$
	d	$\frac{1}{4}$	**e**	$\frac{4}{3}$	**f**	$\frac{16}{9}$
7	**a**	x^{-1}	**b**	x^{-7}	**c**	$x^{\frac{1}{4}}$
	d	$x^{\frac{2}{5}}$	**e**	$x^{-\frac{1}{3}}$	**f**	$x^{-\frac{2}{3}}$
8	**a**	$\frac{1}{x^3}$	**b**	1	**c**	$\sqrt[5]{x}$
	d	$\sqrt[5]{x^2}$	**e**	$\frac{1}{\sqrt{x}}$	**f**	$\frac{1}{\sqrt[4]{x^3}}$
9	**a**	$5x^{\frac{1}{2}}$	**b**	$2x^{-3}$	**c**	$\frac{1}{3}x^{-4}$
	d	$2x^{-\frac{1}{2}}$	**e**	$4x^{-\frac{1}{3}}$	**f**	$3x^0$
10	**a**	$x^3 + x^{-2}$	**b**	$x^3 + x$	**c**	$x^{-2} + x^{-7}$

Section 2 – Factorising and completing the square

Factorising expressions

Key points and examples   Video link

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$ Video link

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

Hint

Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$ **d** $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$ **8** Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Answers

1 **a** $2x^3y^3(3x-5y)$ **b** $7a^3b^2(3b^3+5a^2)$

c $5x^2y^2(5-2x+3y)$

2 **a** $(x+3)(x+4)$ **b** $(x+7)(x-2)$

c $(x-5)(x-6)$ **d** $(x-8)(x+3)$

e $(x-9)(x+2)$ **f** $(x+5)(x-4)$

g $(x-8)(x+5)$ **h** $(x+7)(x-4)$

3 **a** $(6x-7y)(6x+7y)$ **b** $(2x-9y)(2x+9y)$

c $2(3a-10bc)(3a+10bc)$

4 **a** $(x-1)(2x+3)$ **b** $(3x+1)(2x+5)$

c $(2x+1)(x+3)$ **d** $(3x-1)(3x-4)$

e $(5x+3)(2x+3)$ **f** $2(3x-2)(2x-5)$

5 **a** $\dfrac{2(x+2)}{x-1}$ **b** $\dfrac{x}{x-1}$

c $\dfrac{x+2}{x}$ **d** $\dfrac{x}{x+5}$

e $\dfrac{x+3}{x}$ **f** $\dfrac{x}{x-5}$

6 **a** $\dfrac{3x+4}{x+7}$ **b** $\dfrac{2x+3}{3x-2}$

c $\dfrac{2-5x}{2x-3}$ **d** $\dfrac{3x+1}{x+4}$

7 $(x+5)$ **8** $\dfrac{4(x+2)}{x-2}$

Completing the square

Key points and examples **Video link**

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$
$= 2\left(x^2-\frac{5}{2}x\right)+1$	**2** Now complete the square by writing $x^2-\frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right]+1$	
$= 2\left(x-\frac{5}{4}\right)^2 - \frac{25}{8}+1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 – 10x – 3$
c $x^2 – 8x$ **d** $x^2 + 6x$
e $x^2 – 2x + 7$ **f** $x^2 + 3x – 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 – 8x – 16$ **b** $4x^2 – 8x – 16$
c $3x^2 + 12x – 9$ **d** $2x^2 + 6x – 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 – 2x$
c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 **a** $(x + 2)^2 - 1$ **b** $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$ **d** $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$ **f** $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$ **d** $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 **a** $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ **b** $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ **d** $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x + 3)^2 + 3$

Section 3 – Solving and Sketching Quadratics

Solving quadratic equations by factorisation

Key points and examples Video link

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the b term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

Solving quadratic equations by completing the square

Key points and examples [Video link]

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c=0$
$(x + 3)^2 - 9 + 4 = 0$	
$(x + 3)^2 - 5 = 0$	**2** Simplify.
$(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$
$2\left(x^2-\frac{7}{2}x\right)+4=0$	
$2\left[\left(x-\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2\right]+4=0$	**2** Now complete the square by writing $x^2-\frac{7}{2}x$ in the form $\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2$
$2\left(x-\frac{7}{4}\right)^2-\frac{49}{8}+4=0$	**3** Expand the square brackets.
$2\left(x-\frac{7}{4}\right)^2-\frac{17}{8}=0$	**4** Simplify.
	(continued on next page)
$2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$	**6** Divide both sides by 2.
$x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

> **Hint**
>
> Get all terms onto one side of the

Answers

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \dfrac{-3+\sqrt{89}}{10}$ or $x = \dfrac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \dfrac{-3+\sqrt{23}}{2}$ or $x = \dfrac{-3-\sqrt{23}}{2}$

c $x = \dfrac{5+\sqrt{13}}{2}$ or $x = \dfrac{5-\sqrt{13}}{2}$

Solving quadratic equations by using the formula

Key points and examples [Video link]

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \frac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \frac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$

7 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

8 **a** $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$

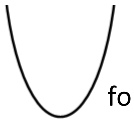
b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

Sketching quadratic graphs

Key points and examples [Video link]

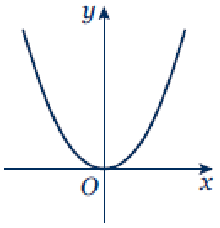
- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.


for $a > 0$

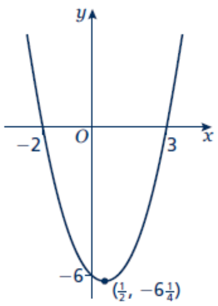
for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	**2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$ So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
	5 $a = 1$ which is greater than zero, so the graph has the shape: 
$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	**6** To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

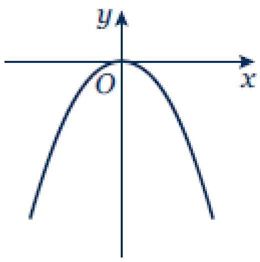
5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

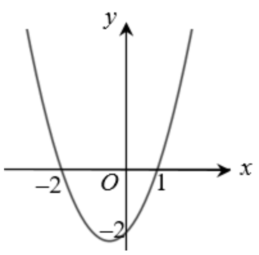
6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

1



2 **a**



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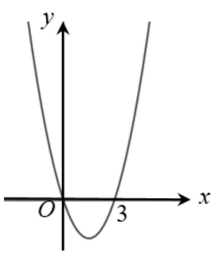

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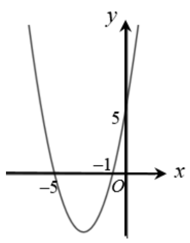

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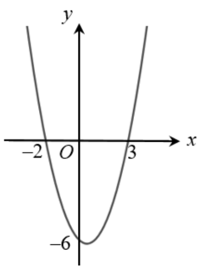

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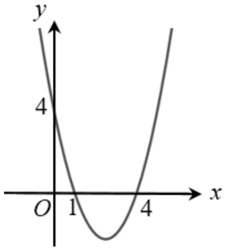

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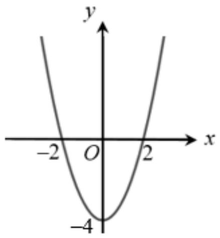

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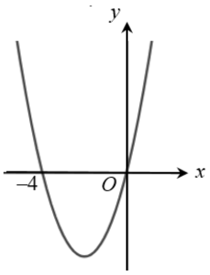

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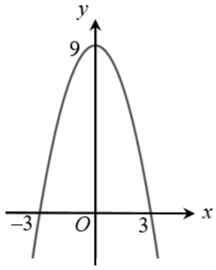

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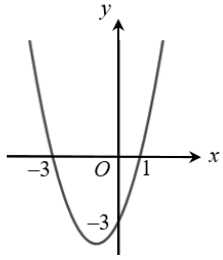

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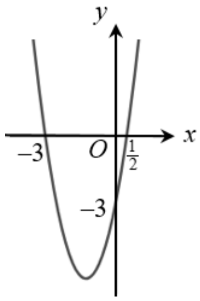
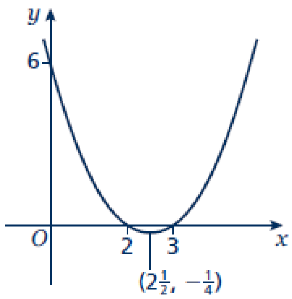


5 **a**



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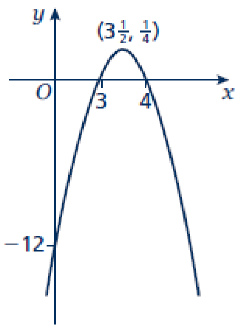

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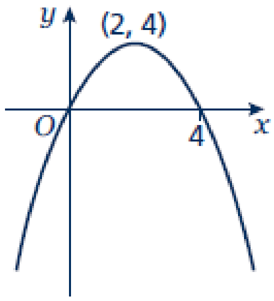

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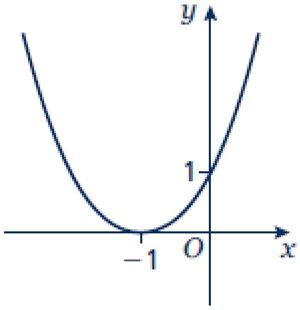


Line of symmetry at $x = -1$.

Section 4 – Simultaneous equations

Solving linear simultaneous equations using the elimination method

Key points and examples   [Video link]

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{rl} & 3x + y = 5 \\ - & x + y = 1 \\ \hline & 2x \quad = 4 \end{array}$ So $x = 2$ Using $x + y = 1$ $2 + y = 1$ So $y = -1$ Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	• **1** Subtract the second equation from the first equation to eliminate the y term. **2** To find the value of y, substitute $x = 2$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = 5 \\ \hline & 6x \quad = 18 \end{array}$ So $x = 3$ Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$ Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**1** Add the two equations together to eliminate the y term. **2** To find the value of y, substitute $x = 3$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x \quad = 28$ So $x = 4$ Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$ Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term. **2** To find the value of y, substitute $x = 4$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

Solving linear simultaneous equations using the substitution method

Key points and examples   Video link

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	**1** Rearrange the first equation.
$4x + 3(2x - 16) = -3$	**2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$ $10x - 48 = -3$	**3** Expand the brackets and simplify.
$10x = 45$ So $x = 4\frac{1}{2}$	**4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

7 $x = 9$, $y = 5$

8 $x = -2$, $y = -7$

9 $x = \frac{1}{2}$, $y = 3\frac{1}{2}$

10 $x = \frac{1}{2}$, $y = 3$

11 $x = -4$, $y = 5$

12 $x = -2$, $y = -5$

13 $x = \frac{1}{4}$, $y = 1\frac{3}{4}$

14 $x = -2$, $y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}$, $y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

Key points and examples

Video link

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Answers

1 $x = 1, y = 3$ $\quad x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$ $\quad x = 4, y = 2$

3 $x = 1, y = -2$ $\quad x = 2, y = -1$

4 $x = 4, y = 1$ $\quad x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$ $\quad x = 2, y = 1$

6 $x = 7, y = 2$ $\quad x = -1, y = -6$

7 $x = 0, y = 5$ $\quad x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$ $\quad x = 3, y = 5$

9 $x = -2, y = -4$ $\quad x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$ $\quad x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$ $\quad x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$ $\quad x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Section 5 – Inequalities and rearranging equations

Linear inequalities

Key points and examples

Video link

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1 **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$

d $x > -\frac{7}{2}$ **e** $x \geq 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$

3 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

4 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

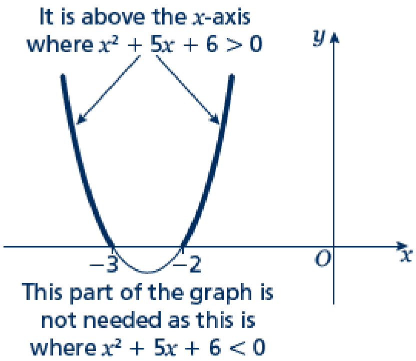
6 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

Key points and examples [Video link]

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

<table>
<tr><td>$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or $x = -2$


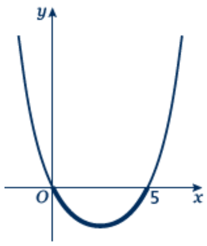
$x < -3$ or $x > -2$</td>
<td>**1** Solve the quadratic equation by factorising.

2 Sketch the graph of $y = (x + 3)(x + 2)$

3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$

4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$</td></tr>
</table>

Example 2 Find the set of values of x which satisfy $x^2 - 5x \le 0$

<table>
<tr><td>$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x = 5$


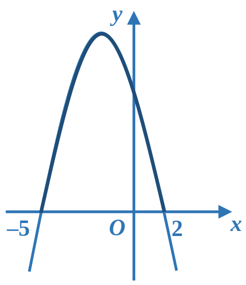
$0 \le x \le 5$</td>
<td>**1** Solve the quadratic equation by factorising.

2 Sketch the graph of $y = x(x - 5)$

3 Identify on the graph where $x^2 - 5x \le 0$, i.e. where $y \le 0$

4 Write down the values which satisfy the inequality $x^2 - 5x \le 0$</td></tr>
</table>

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \ge 0$

<table>
<tr><td>$-x^2 - 3x + 10 = 0$
$(-x + 2)(x + 5) = 0$
$x = 2$ or $x = -5$


$-5 \le x \le 2$</td>
<td>**1** Solve the quadratic equation by factorising.

2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$

3 Identify on the graph where $-x^2 - 3x + 10 \ge 0$, i.e. where $y \ge 0$

4 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \ge 0$</td></tr>
</table>

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Rearranging equations

Key points and examples [Video link 1] [Video link 2]

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$	**1** Get the terms containing t on one side and everything else on the other side.
$v - u = at$ $t = \dfrac{v-u}{a}$	**2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$	**1** All the terms containing t are already on one side and everything else is on the other side.
$r = t(2 - \pi)$	**2** Factorise as t is a common factor.
$t = \dfrac{r}{2-\pi}$	**3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\dfrac{t+r}{5} = \dfrac{3t}{2}$.

$\dfrac{t+r}{5} = \dfrac{3t}{2}$	**1** Remove the fractions first by multiplying throughout by 10.
$2t + 2r = 15t$ $2r = 13t$	**2** Get the terms containing t on one side and everything else on the other side and simplify.
$t = \dfrac{2r}{13}$	**3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t - 1) = 3t + 5$	**2** Expand the brackets.
$rt - r = 3t + 5$ $rt - 3t = 5 + r$	**3** Get the terms containing t on one side and everything else on the other side.
$t(r - 3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \dfrac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \dfrac{S}{T}$ [T]

4 $p = \dfrac{q-r}{t}$ [t]

5 $u = at - \dfrac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y]

8 $x = \dfrac{2a-1}{3-a}$ [a]

9 $x = \dfrac{b-c}{d}$ [d]

10 $h = \dfrac{7g-9}{2+g}$ [g]

11 $e(9 + x) = 2e + 1$ [e]

12 $y = \dfrac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\dfrac{xy}{z} = \dfrac{ab}{cd}$ **b** $\dfrac{4\pi cx}{d} = \dfrac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac\cos B$.

Extend

17 Make x the subject of the following equations.

a $\dfrac{p}{q}(sx+t) = x-1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

Answers

1 $d = \dfrac{C}{\pi}$

2 $w = \dfrac{P-2l}{2}$

3 $T = \dfrac{S}{D}$

4 $t = \dfrac{q-r}{p}$

5 $t = \dfrac{2u}{2a-1}$

6 $x = \dfrac{V}{a+4}$

7 $y = 2 + 3x$

8 $a = \dfrac{3x+1}{x+2}$

9 $d = \dfrac{b-c}{x}$

10 $g = \dfrac{2h+9}{7-h}$

11 $e = \dfrac{1}{x+7}$

12 $x = \dfrac{4y-3}{2+y}$

13 **a** $r = \sqrt{\dfrac{A}{\pi}}$ **b** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$ **c** $r = \dfrac{P}{\pi+2}$ **d** $r = \sqrt{\dfrac{3V}{2\pi h}}$

14 **a** $x = \dfrac{abz}{cdy}$ **b** $x = \dfrac{3dz}{4\pi cpy^2}$

15 $\sin B = \dfrac{b\sin A}{a}$

16 $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

17 **a** $x = \dfrac{q+pt}{q-ps}$ **b** $x = \dfrac{3py+2pqy}{3p-apq} = \dfrac{y(3+2q)}{3-aq}$

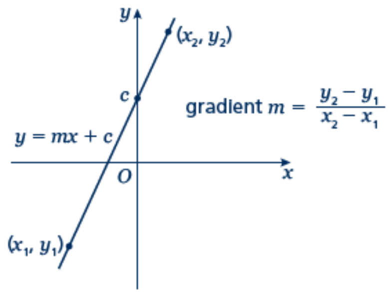
Section 6 – Linear graphs

Straight line graphs

Key points and examples

[Video link]

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient = $m = \frac{2}{3}$ y-intercept = $c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	• **1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.

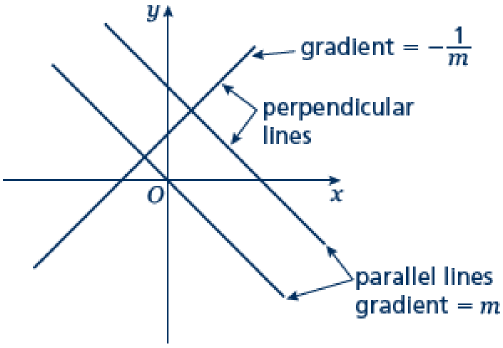
The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

Key points and examples Video link

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates into the equation $y = 2x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. **2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	• **1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

Hint
If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$, $y = 2x - 7$ **b** $y = 3x$, $2x + y - 3 = 0$ **c** $y = 4x - 3$, $4y + x = 2$

d $3x - y + 5 = 0$, $x + 3y = 1$ **e** $2x + 5y - 1 = 0$, $y = 2x + 7$ **f** $2x - y = 6$, $6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

A Level Mathematics Summer assignment

You will be asked to submit solutions (on paper) to these questions in the first week of the course.
You will be tested on these skills around 3 weeks into the course.

Section A

1 Simplify these expressions.

a $\dfrac{x^3 \times x^4}{x^2}$ **(1 mark)**

b $(2x^3)^4$ **(1 mark)**

c $\dfrac{9x^{\frac{1}{2}}}{(27x^{-2})^{\frac{2}{3}}}$ **(3 marks)**

2 Solve $2x^2 \times 4x^4 = 512$ **(2 marks)**

3 Find the value of *x*.

$x^{-\frac{4}{3}} = \dfrac{1}{256}$ **(2 marks)**

1,2,3 video link

4 **a** Write $\sqrt{240}$ in the form $a\sqrt{15}$, where *a* is an integer. **(1 mark)**

b Expand and simplify $(2-\sqrt{3})(5+2\sqrt{3})$. **(2 marks)**

c Simplify $\dfrac{2+\sqrt{5}}{3-\sqrt{5}}$ giving your answer in the form $a+b\sqrt{c}$, where *a*, *b* and *c* are rational numbers. **(3 marks)**

5 The area of a triangle is given as $(7+3\sqrt{3})$ cm^2.

The base of the triangle is $(5-\sqrt{3})$ cm, and the perpendicular height is $(p+q\sqrt{3})$ cm.

Find the values of *p* and *q*. **(4 marks)**

4,5 video link

6 Expand and simplify these expressions.

a $3(x-2y)$ **(1 mark)**

b $(2x-3)(3x+5)$ **(2 marks)**

c $(x-2)^2(x+5)$ **(3 marks)**

7 Fully factorise these expressions.

a $2xy-4x$ **(1 mark)**

b x^2+2x-3 **(1 mark)**

6,7 video link

8 Solve these equations.

a $3x-7=17$ **(1 mark)**

b $x^2-6x+5=0$ **(2 marks)**

c $2x^2-5x+1=0$ **(2 marks)**

8b,c video link

9 Solve these pairs of simultaneous equations.

a $2x+y=7$ **(3 marks)**

$3x-y=8$

b $y=3x-1$ **(3 marks)**

$3y=6x+1$

c $2x-y=9$ **(4 marks)**

$x^2+y^2=17$

9a,b video link

9b,c video link

10 Solve these inequalities.

a $7x-6\leqslant 8$ **(1 mark)**

b $3x+2\geqslant 7x-4$ **(2 marks)**

c $x^2+12x-28>0$ **(2 marks)**

10a,b video link

10c video link

11 The function f is defined as $\mathrm{f}(x)=5x+2$

Find the value of $\mathrm{f}(-4)$. **(1 mark)**

11 video link

Section B

1 Simplify these expressions as far as possible.

a $\dfrac{x^2-2x-3}{x^2+2x+1}$ **(3 marks)**

b $\dfrac{x^2-25}{x^2+6x+8}\div\dfrac{x^2-2x-15}{x^2-16}$ **(4 marks)**

1a video link

1b video link

2 The line *l* is a tangent to the circle $x^2 + y^2 = 20$ at the point *P*(2, 4).

The tangent intersects the *y*-axis at point *A*. Find the area of the triangle *OPA*. **(5 marks)**

2 video link **(Just a hint)**

3 Expand and simplify $\left(\sqrt{p} + 2\sqrt{q}\right)\left(2\sqrt{p} - \sqrt{q}\right)$ **(3 marks)**

3 video link

4 **a** Write $3x^2 - 12x + 7$ in the form $a(x+b)^2 + c$ **(3 marks)**

b Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 3x^2 - 12x + 7$ **(1 mark)**

4 video link

5 Prove algebraically that the product of three consecutive **odd** numbers is always an odd number. **(4 marks)**

5 video link

6 The functions g and f are defined as $\mathrm{g}(x) = \dfrac{2x}{4-x}$ and $\mathrm{f}(x) = 3x - 1$

Given that $x \neq 4$, find the value(s) of *x* such that $\mathrm{g}(x) = \mathrm{f}(x)$, giving your answer(s) to 2 decimal places. **(6 marks)**

6 video link

7 The line l_1 has equation $y = -\frac{1}{2}x + 3$ and intersects the *x*- and *y*-axes at the points *A* and *B* respectively.

a Find the exact length of the line segment *AB*. **(3 marks)**

b Find the equation of the line l_2 perpendicular to l_1 which passes through the point *P*(−1, −2). **(2 marks)**

The line l_2 intersects l_1 at the point *C*.

c Find the midpoint of the line segment *AC*. **(4 marks)**

7 video link **(look for more links on this page)**

8 A triangle *ABC* has side lengths *AB* = 10 cm, *BC* = 15 cm and *AC* = 8 cm.

a Find the size of the largest angle, giving your anwer to 2 decimal places. **(3 marks)**

b Find the area of the triangle, giving your anwer to 2 decimal places. **(2 marks)**

8a video link

8b video link

9 **a** Sketch the graph of *y* = cos *x* for $-180 \leqslant x \leqslant 360°$, showing the points where the graph cuts the axes. **(2 marks)**

b Hence find the exact values of *x* in the interval $-180 \leqslant x \leqslant 360°$ for which

cos $x = -\dfrac{\sqrt{3}}{2}$ **(3 marks)**

9a video link

9b video link

Suggested preparation work

Sometimes we learn Mathematics to allow us to solve real world problems, sometimes to explore the complexities of the natural structure of the world around us and sometimes just for fun. Here is a small collection of optional content for you to explore Mathematics in a wider context, there is a lot of great content out there so let us know if you find something exciting you want to share!

Documentaries, videos and talks

Magic Numbers: Hannah Fry's Mysterious World of Maths

A three part documentary series exploring the mysteries of mathematics.

https://www.youtube.com/watch?v=cyvDG8qjt-M

Numberphile

https://www.youtube.com/channel/UCoxcjq-8xIDTYp3uz647V5A

A whole channel devoted to sharing mathematical ideas.

TED Talks

https://www.ted.com/playlists/189/math_talks_to_blow_your_mind

You must NEVER use the word 'math' but they are forgiven as this list includes a talk by Benoit Mandelbrot, the father of fractals, and Marcus du Sautoy talking about symmetry.

Podcasts

More or Less https://www.bbc.co.uk/sounds/brand/b006qshd

Tim Harford explains - and sometimes debunks - the numbers and statistics used in political debate, the news and everyday life

Books

Simon Singh **The Simpsons and their Mathematical Secrets**

This is a bit more light-hearted but has some good maths in it. The chapters on Futurama (made by the same people as the Simpsons) are particularly good - one writer proved a new theorem especially for the plot an episode!

Hannah Fry **Hello World**

How to be human in the age of the machine – this book explores algorithms in the world around us.

Marcus du Sautoy **The Music of the Primes**

Du Sautoy has the ability to explain complex ideas simply. This book, about the building blocks of mathematics and Number Theory, also talks about the tantalising subject of unsolved problems in mathematics.

Films

A Beautiful Mind	The story of game theorist John Nash
Good Will Hunting	Mathematics is more of a back drop in this one but still a good film
Hidden Figures	The story of the NASA mathematicians who helped launch John Glenn into space
The Imitation Game	The story of how cryptologist Alan Turing cracked the enigma code, shortening the second world war, including King Ecgbert alumnus, Matthew Beard
The Man who Knew Infinity	The story of self-taught mathematician Ramanujan who collaborated with GH Hardy to produce some of the strangest results in Number Theory
The Theory of Everything	Stephen Hawking may be more of a physicist than a mathematician, but it’s a wonderful film, and the list would be incomplete without it